To whom it may concern!

SOME USEFUL ELECTRIC CIRCUITS

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1. The twin-T Bridge

The twin-T bridge shown in *Fig. 1* is frequently used as a feedback element in selective amplifiers, oscillators and for many other purposes. It consists of two T-circuits connected in parallel. The analysis of this circuit is best carried out by transforming both T into equivalent Π-connection and connecting them parallel as shown in *Fig. 2*, where

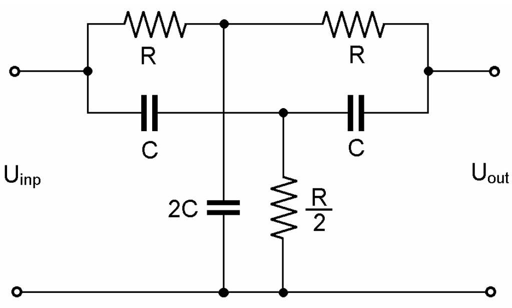

Fig. 1

$$\boldsymbol{Z}_A = 2(R + \mathrm{j}\omega CR), \tag{1}$$

$$\boldsymbol{Z}'_A = 2\left(\frac{1}{\mathrm{j}\omega C} - \frac{1}{\omega^2 C^2 R}\right), \tag{2}$$

$$\boldsymbol{Z}_B = \boldsymbol{Z}'_B = \boldsymbol{Z}_C = \boldsymbol{Z}'_C = R + \frac{1}{\mathrm{j}\omega C}, \tag{3}$$

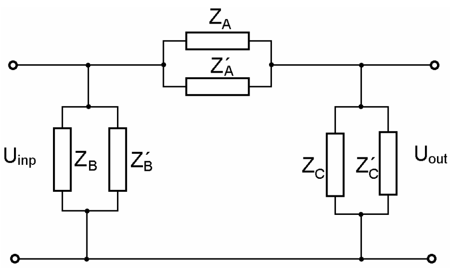

Fig. 2

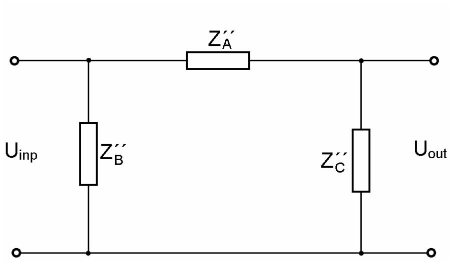

Fig. 3

(such transfiguration will be analysed in the article "Delta-star transformation").

Adding the impedances in *Fig. 2* in parallel we get a new circuit shown in *Fig. 3*, where

$$\boldsymbol{Z}''_A = 2R\frac{1+\mathrm{j}\omega CR}{1-\omega^2C^2R^2}, \tag{4}$$

$$\boldsymbol{Z}''_B = \boldsymbol{Z}''_C = \frac{1}{2}\left(R+\frac{1}{\mathrm{j}\omega C}\right), \tag{5}$$

The complex transmission coefficient is

$$\boldsymbol{K}(\omega) = \frac{\boldsymbol{U}_{out}(\omega)}{\boldsymbol{U}_{inp}(\omega)} = \frac{\boldsymbol{Z}''_C}{\boldsymbol{Z}''_A+\boldsymbol{Z}''_C} = \frac{1-\omega^2C^2R^2}{1-\omega^2C^2R^2+\mathrm{j}4\omega CR}. \tag{6}$$

The absolute value of the transmission coefficient is given by

$$K(\omega) = \frac{U_{out}(\omega)}{U_{inp}(\omega)} = \left|\frac{\boldsymbol{Z}''_C}{\boldsymbol{Z}''_A+\boldsymbol{Z}''_C}\right| = \frac{1-\omega^2C^2R^2}{\sqrt{\left(1-\omega^2C^2R^2\right)^2+\left(4\omega CR\right)^2}} =$$

$$= \frac{1-\left(\dfrac{\omega}{\omega_0}\right)^2}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_0}\right)^2\right]^2+\left(4\dfrac{\omega}{\omega_0}\right)^2}}. \tag{7}$$

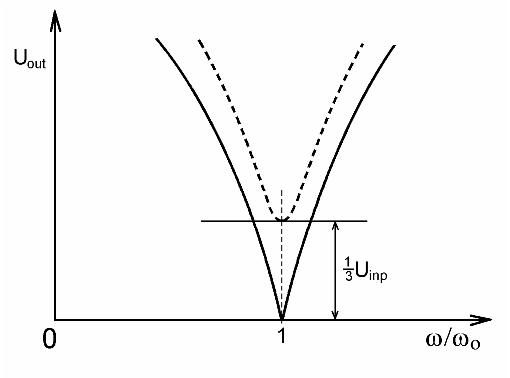

Fig. 4

where $\omega_0 = 1/(RC)$. If the resistors and capacitors in *Fig. 1* are fixed, the output voltage is dependent on the frequency of the input voltage. The dependence of $U_{out}(\omega/\omega_0)$ is shown in *Fig. 4*. We see, that there is a single frequency

$$\omega_0 = \frac{1}{RC}, \tag{8a}$$

$$f_0 = \frac{1}{2\pi RC}, \tag{8b}$$

at which the output voltage is zero. In the vicinity of this frequency the circuit behaves itself as a resonant circuit with relatively high Q-factor. The circuit is particularly useful at low frequencies, where the equivalent RLC-circuit request large values of *L* and *C*.

Another way to analyse the twin-T bridge is using the method of node voltages.

2. The bridged T

If we remove the capacitor 2*C* in the circuit of *Fig. 1*, we get a new selective element, commonly called the bridged T-filter, shown in *Fig. 5*. The analysis similar to this used in preceding case leads to an equivalent Π-connection (see *Fig. 3*), with

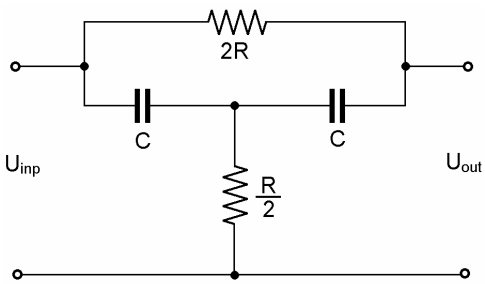

Fig. 5

$$\boldsymbol{Z}''_A = 2\frac{R + \mathrm{j}\omega C}{1-\omega^2 C^2 R^2 + \mathrm{j}\omega CR}, \tag{9}$$

$$\boldsymbol{Z}''_B = \boldsymbol{Z}''_C = \left(R + \frac{1}{\mathrm{j}\omega C}\right), \tag{10}$$

The complex transmission coefficient is

$$\boldsymbol{K}(\omega) = \frac{\boldsymbol{U}_{out}(\omega)}{\boldsymbol{U}_{inp}(\omega)} = \frac{\boldsymbol{Z}''_C}{\boldsymbol{Z}''_A + \boldsymbol{Z}''_C} = \frac{(\omega CR)^4 + (\omega CR)^2 + 1 + \mathrm{j}2\omega CR\left[(\omega CR)^2 - 1\right]}{(\omega CR)^4 + 7(\omega CR)^2 + 1} =$$

$$= \frac{\left(\frac{\omega}{\omega_0}\right)^4 + \left(\frac{\omega}{\omega_0}\right)^2 + 1 + \mathrm{j}2\frac{\omega}{\omega_0}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]}{\left(\frac{\omega}{\omega_0}\right)^4 + 7\left(\frac{\omega}{\omega_0}\right)^2 + 1}. \tag{11}$$

where

$$\omega_0 = \frac{1}{RC}, \tag{12}$$

From the expression (11) is obvious, that the output voltage is real if $\omega = \omega_0 = 1/(RC)$, and at this frequency approaches minimum, which is

$$U_{out.min} = \frac{1}{3} U_{inp}. \tag{13}$$

The dependence $U_{out}(\omega/\omega_0)$ is shown in *Fig. 4* (dashed curve).

3. Delta-star transformation

The passive three terminal network consisting of three impedances $\boldsymbol{Z}_A$, $\boldsymbol{Z}_B$ and $\boldsymbol{Z}_C$ as shown in *Fig. 1a*, is said to form a delta (Δ) – connection. The passive three terminal network consisting of three impedances $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$ and $\boldsymbol{Z}_3$ as shown in *Fig. 1b*, is said to form a star (Y) – connection. The two circuits are equivalent if their respective input, output and transfer impedance are equal.

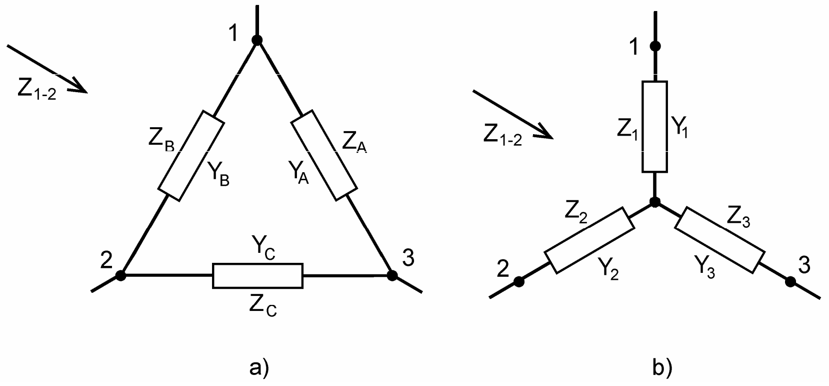

Fig. 1

Assuming open circuit conditions, we get from *Figs. 1a, 1b*:

Impedance		Delta		Star
$\boldsymbol{Z}_{1-2}$	$=$	$\dfrac{\boldsymbol{Z}_B(\boldsymbol{Z}_A+\boldsymbol{Z}_C)}{\boldsymbol{Z}}$	$=$	$\boldsymbol{Z}_1+\boldsymbol{Z}_2$
$\boldsymbol{Z}_{2-3}$	$=$	$\dfrac{\boldsymbol{Z}_C(\boldsymbol{Z}_A+\boldsymbol{Z}_B)}{\boldsymbol{Z}}$	$=$	$\boldsymbol{Z}_2+\boldsymbol{Z}_3$
$\boldsymbol{Z}_{3-1}$	$=$	$\dfrac{\boldsymbol{Z}_A(\boldsymbol{Z}_B+\boldsymbol{Z}_C)}{\boldsymbol{Z}}$	$=$	$\boldsymbol{Z}_1+\boldsymbol{Z}_3$

where

$$\boldsymbol{Z} = \boldsymbol{Z}_A + \boldsymbol{Z}_B + \boldsymbol{Z}_C .$$

Rearranging the above equations gives

$$\boldsymbol{Z}_1 + \boldsymbol{Z}_2 = \frac{\boldsymbol{Z}_A \boldsymbol{Z}_B}{\boldsymbol{Z}} + \frac{\boldsymbol{Z}_B \boldsymbol{Z}_C}{\boldsymbol{Z}}, \tag{1}$$

$$\boldsymbol{Z}_2 + \boldsymbol{Z}_3 = \frac{\boldsymbol{Z}_B \boldsymbol{Z}_C}{\boldsymbol{Z}} + \frac{\boldsymbol{Z}_C \boldsymbol{Z}_A}{\boldsymbol{Z}}, \tag{2}$$

$$\boldsymbol{Z}_3 + \boldsymbol{Z}_1 = \frac{\boldsymbol{Z}_C \boldsymbol{Z}_A}{\boldsymbol{Z}} + \frac{\boldsymbol{Z}_A \boldsymbol{Z}_B}{\boldsymbol{Z}}. \tag{3}$$

Substracting Eq. (2) from Eq. (1)

$$\boldsymbol{Z}_1 - \boldsymbol{Z}_3 = \frac{\boldsymbol{Z}_A \boldsymbol{Z}_B}{\boldsymbol{Z}} - \frac{\boldsymbol{Z}_C \boldsymbol{Z}_A}{\boldsymbol{Z}}, \tag{4}$$

Adding Eq. (3) and Eq. (4) gives

$$\boldsymbol{Z}_1 = \frac{\boldsymbol{Z}_A \boldsymbol{Z}_B}{\boldsymbol{Z}}, \tag{4}$$

similarly

$$\boldsymbol{Z}_2 = \frac{\boldsymbol{Z}_B \boldsymbol{Z}_C}{\boldsymbol{Z}}, \tag{6}$$

and

$$\boldsymbol{Z}_3 = \frac{\boldsymbol{Z}_C \boldsymbol{Z}_A}{\boldsymbol{Z}}. \tag{7}$$

The reverse transformation of "star network" into "delta" is best carried out by using impedances replaced by admittances, and short circuiting one pair of corresponding terminals in each network at a time. Thus from *Figs. 1a, 1b* we get:

Short - circuited terminals		Delta		Star
$1-3$	$\boldsymbol{Y}_{1-2}$	$= \boldsymbol{Y}_B + \boldsymbol{Y}_C$	$=$	$\dfrac{\boldsymbol{Y}_2(\boldsymbol{Y}_3 + \boldsymbol{Y}_1)}{\boldsymbol{Y}}$
$2-3$	$\boldsymbol{Y}_{1-3}$	$= \boldsymbol{Y}_A + \boldsymbol{Y}_B$	$=$	$\dfrac{\boldsymbol{Y}_1(\boldsymbol{Y}_2 + \boldsymbol{Y}_3)}{\boldsymbol{Y}}$
$1-2$	$\boldsymbol{Y}_{2-3}$	$= \boldsymbol{Y}_C + \boldsymbol{Y}_A$	$=$	$\dfrac{\boldsymbol{Y}_3(\boldsymbol{Y}_1 + \boldsymbol{Y}_2)}{\boldsymbol{Y}}$

where

$$\boldsymbol{Y} = \boldsymbol{Y}_1 + \boldsymbol{Y}_2 + \boldsymbol{Y}_3.$$

Solving for "delta" impedances:

$$\boldsymbol{Y}_A = \frac{\boldsymbol{Y}_3 \boldsymbol{Y}_1}{\boldsymbol{Y}}, \tag{8}$$

$$\boldsymbol{Y}_B = \frac{\boldsymbol{Y}_1 \boldsymbol{Y}_2}{\boldsymbol{Y}}, \tag{9}$$

$$\boldsymbol{Y}_C = \frac{\boldsymbol{Y}_2 \boldsymbol{Y}_3}{\boldsymbol{Y}}, \tag{10}$$

in terms of impedances

$$\boldsymbol{Z}_A = \boldsymbol{Z}_3 + \boldsymbol{Z}_1 + \frac{\boldsymbol{Z}_3\boldsymbol{Z}_1}{\boldsymbol{Z}_2}, \tag{11}$$

$$\boldsymbol{Z}_B = \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_3}, \tag{12}$$

$$\boldsymbol{Z}_C = \boldsymbol{Z}_2 + \boldsymbol{Z}_3 + \frac{\boldsymbol{Z}_2\boldsymbol{Z}_3}{\boldsymbol{Z}_1}. \tag{13}$$